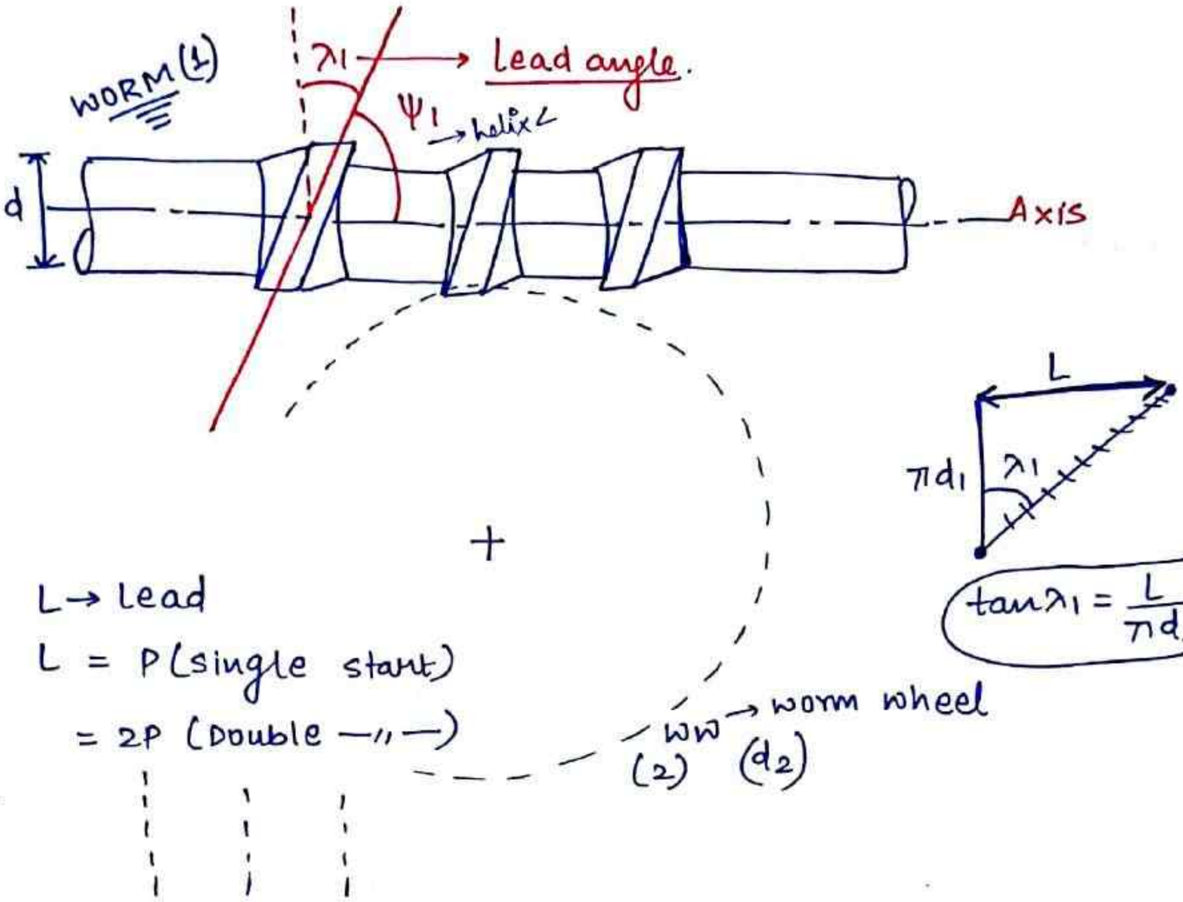
Worm and Worm wheel :-
Terminology of

WORM → 1

WORM WHEEL → 2

($\theta = 90^\circ$)



$$\tan\lambda_1 = \frac{L}{\pi d_1} \quad *$$

L → lead

L = P (single start)

= 2P (Double —"—)

$$\psi_1 + \lambda_1 = 90^\circ$$

$$\psi_1 = (90^\circ - \lambda_1) \quad *$$

$$\theta = (\psi_1 + \psi_2)$$

$$\cancel{90^\circ} = (\cancel{90^\circ} - \lambda_1) + \psi_2$$

$$\psi_2 = \lambda_1 \quad **$$

Centre distance

$\hookrightarrow r_1 + r_2$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_2}{2}\left[\frac{m_1}{m_2} \cdot T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\frac{\cancel{m_n}}{\cos\psi_1}}{\frac{\cancel{m_n}}{\cos\psi_2}} \cdot T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90^\circ - \lambda_1)} \cdot T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[T_1 \cdot \cot\lambda_1 + T_2\right]$$

VR :- $VR = \frac{\omega_2}{\omega_1} = \frac{(L/r_2)}{2\pi}$

$$VR = \frac{L}{\pi(2r_2)} = \frac{L}{\pi d_2}$$ **

Efficiency :-

$$\eta = \frac{\cos(\lambda_1 + \phi)}{\cos(90^\circ - (\lambda_1 + \phi))} \cdot \frac{\cos(90^\circ - \lambda_1)}{\cos\lambda_1}$$

$$\boxed{\eta = \frac{\tan\lambda_1}{\tan(\lambda_1 + \phi)}}$$ **.

Max. Efficiency :-

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}$$ *

07/12/2016

CH # 5

MECHANICAL VIBRATIONS

$\Downarrow$

$\underbrace{\text{To and Fro Periodic motions about their eqm. Positions.}}$

$\underbrace{\text{Harmonic motions.}}$

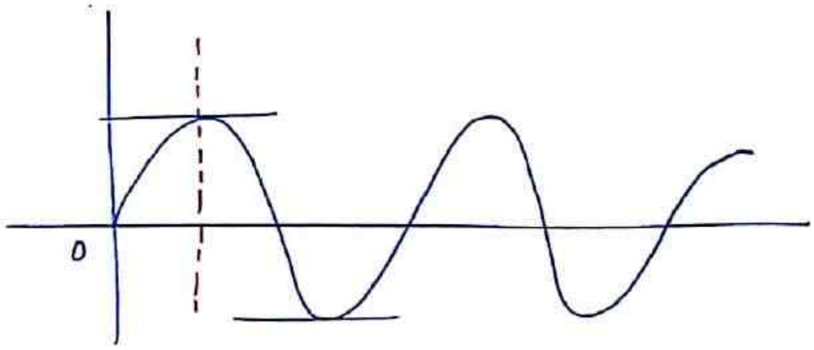
oscillations.

fast

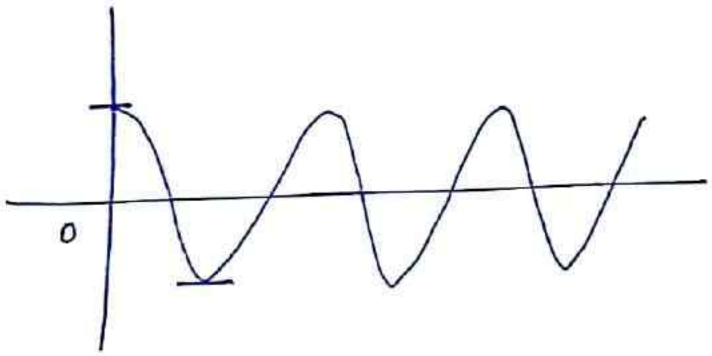
Mean Position
eqm. Position
Zero Position

same.

sin wave → VIB.



cos wave → VIB.



e^x wave
$\log x$ wave
] → No VIB.

Any Vib^r. System is a combination of:

1. Mass : m
(KE storing device)

2. stiffness : s ⟶ depend on (i) material (ii) Geometrical dimensions
(spring const.)
(P.E storing device)

3. Damping
(Energy loss due to kinetic friction)

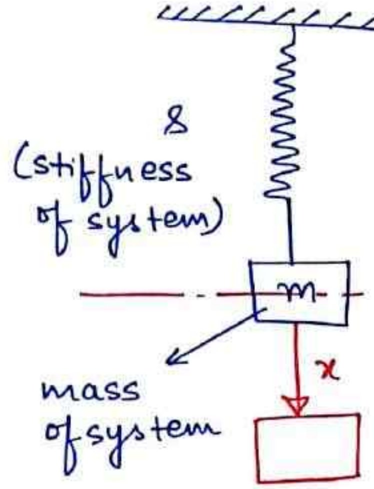
4. $F_{UN} \neq 0$
↳ $\boxed{\text{unbalanced Forces}}$

That vib. system which is not having any Rec./Rot. masses.
Here vib. are introduced due to some external disturbance given initially.

That vib. system which is having Reci./Rot. mass.
↓
Running M/c

*** Natural Vibrations :-

"The vibrations in which there is no kinetic friction at all as well as there is no external force after the initial release of the system are known as Natural Vibrations."



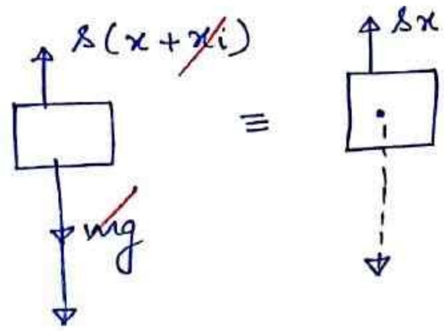
$\boxed{mg = s x_i}$

eq^m. eq^n **.

$x = f(t) = ??$

At t = t

At t = t system (F.B.D.)



NSL :-

$0 - sx = ma$

$\boxed{ma + sx = 0}$ **

$ma + sx = 0$

$m\ddot{x} + sx = 0$

$\boxed{\ddot{x} + \left(\dfrac{s}{m}\right)x = 0}$ *

eq^n. of Natural system.

The sol^n. of this eq^n. is

$\boxed{x = R \sin\left(\sqrt{\dfrac{s}{m}}\, t + \phi\right)}$ *

$R, \phi \to$ const.

Amplitude ⇓ const.

VIB with Freq.

$\boxed{\omega_n = \sqrt{\dfrac{s}{m}}}$ * $\dfrac{\text{rad}}{\text{s}}$

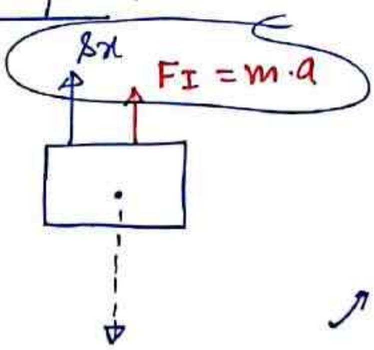
$T_n = \dfrac{2\pi}{\omega_n} \Rightarrow (\text{sec})$

$f_n = \dfrac{\omega_n}{2\pi}$ (Hz)

D'Alembert Principle :-

(DAP).

At t = t system F.B.D.



(DAP) :-

$\boxed{ma + sx = 0}$

**

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$R, \phi \to$ const

are found by Initial condition:-
($t = 0$)

1. At $t = 0$ $\begin{cases} x = x_0 \\ \dot{x} = 0 \end{cases}$ $R = ?$, $\phi = ?$

2. At $t = 0$ $\begin{cases} x = 0 \\ \dot{x} = v_0 \end{cases}$ $R = ?$, $\phi = ?$

3. At $t = 0$ $\begin{cases} x = x_0 \\ \dot{x} = v_0 \end{cases}$ $R = ?$, $\phi = ?$

Finally

eqn. of Natural Vibr. is

$$\ddot{x} + (\omega_n^2)x = 0 \quad **$$

Pb 20marks The Nat. vibr. system eqn. is:-

$$5\ddot{x} + 3x = 0$$

find $\omega_n = ?$

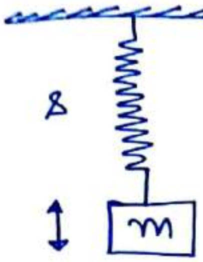
$$\ddot{x} + (3/5)x = 0$$

$$\omega_n^2 = 3/5$$

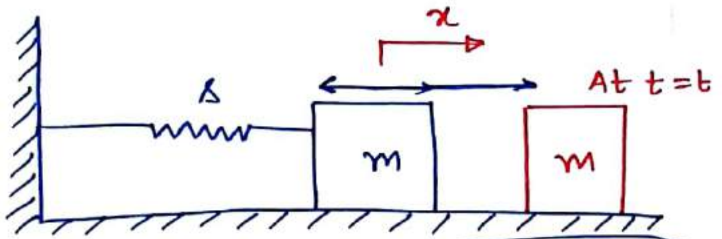
$$\omega_n^2 = 3/5$$

$$\omega_n = \sqrt{3/5} \text{ rad/s} \quad **$$

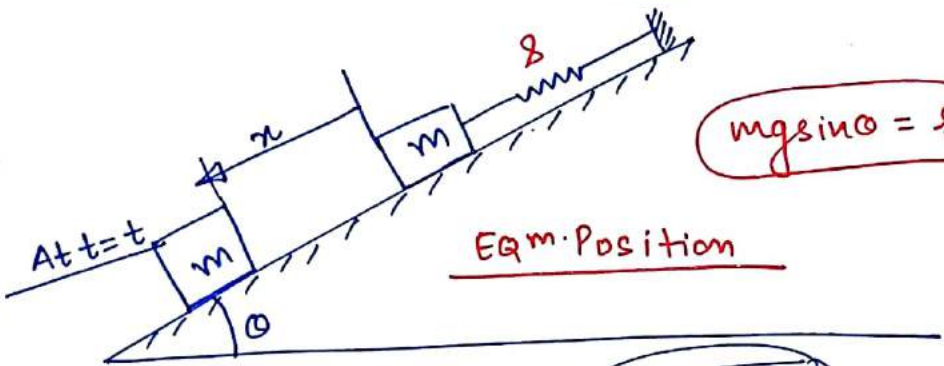
Note :-



$$\omega_n = \sqrt{\frac{s}{m}} \quad *$$



$$\omega_n = \sqrt{\frac{s}{m}} \quad *$$

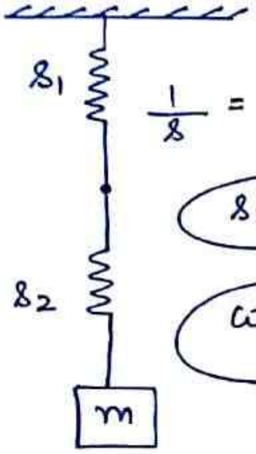


$$mg\sin\theta = s x_i \quad *$$

$$\omega_n = \sqrt{\frac{s}{m}} \quad **$$

Combinations of springs :-

Series :-

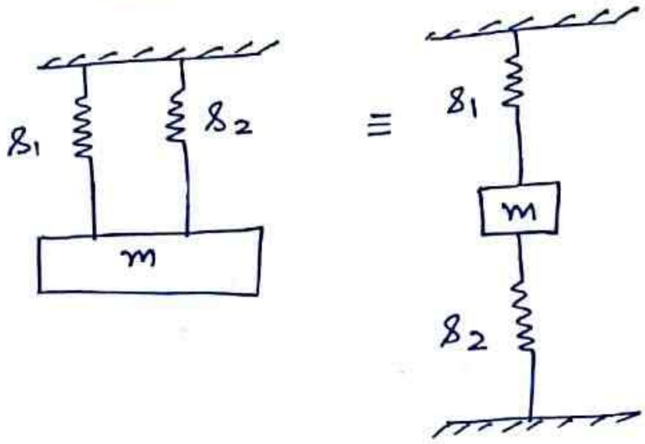


$$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$$

$s_2 = ?$ *

$\omega_n = \sqrt{\frac{s}{m}}$ *

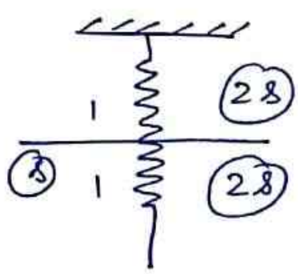
Parallel :-

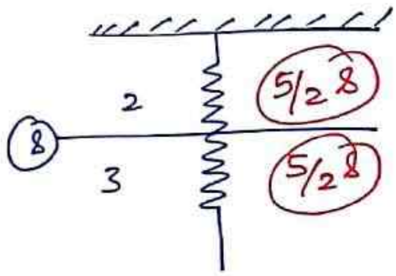


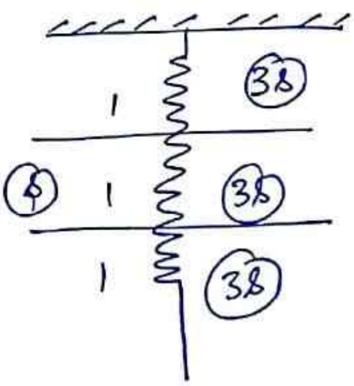
$s = s_1 + s_2$ *

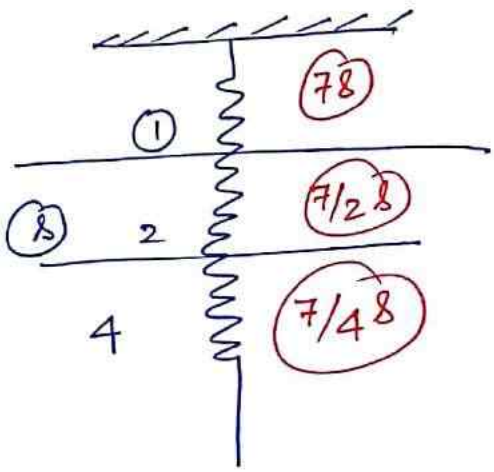
$\omega_n = \sqrt{\frac{s}{m}}$ **

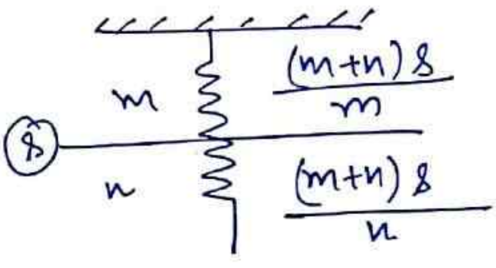
Cutting of springs :-











Energy Method :-

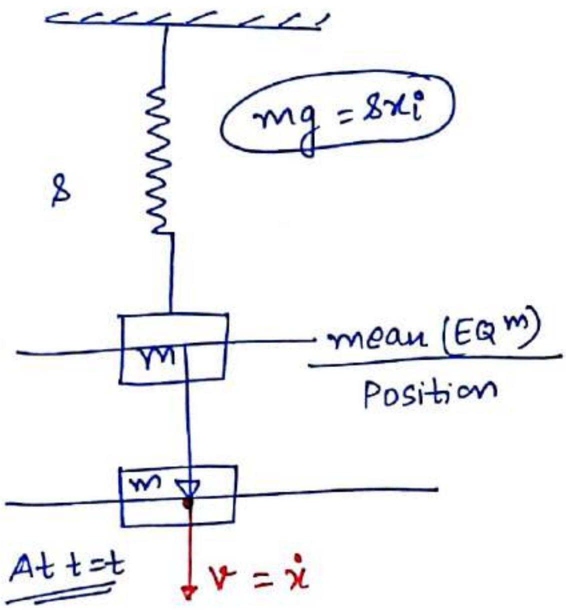
In Nat. Vibr.

Kinetic friction = 0

Energy loss = 0

Total Energy (E) = const.

$$\frac{dE}{dt} = 0 \quad **$$



At t=t

system energy

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2$$

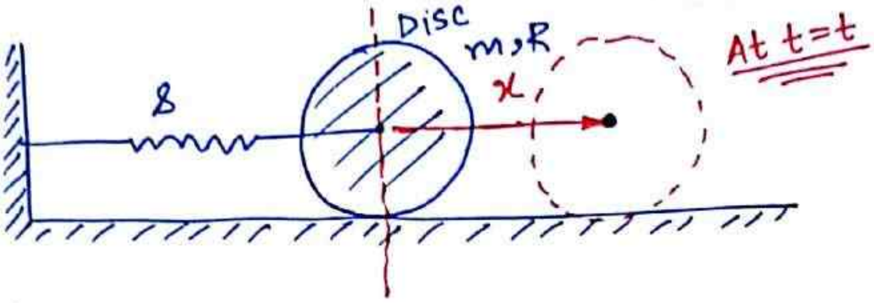
$$dE/dt = \cancel{\tfrac{1}{2}} \cdot s \cdot \cancel{2} x \, dx/dt + \cancel{\tfrac{1}{2}} \cdot m \cdot \cancel{2} v \cdot \frac{dv}{dt} = 0$$

$$m\ddot{x} + sx = 0$$

$$\ddot{x} + \left(s/m\right) x = 0 \quad *$$

$$\omega_n = \sqrt{s/m} \ \text{rad/s} \quad **$$

Pb
2 marks



At t = t

System Energy

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{2} \frac{m \cancel{R^2}}{2} \cdot \frac{v^2}{\cancel{R^2}}$$

$$E = \frac{1}{2} s x^2 + \frac{1}{2} \left(\frac{3m}{2}\right) v^2 \quad **$$

$$\omega_n = \sqrt{\frac{s}{\frac{3m}{2}}} = \sqrt{\frac{2s}{3m}} \quad *$$

Note :-

Disc / Solid cyl] $I = \frac{mR^2}{2}$

Solid sp. $I = \frac{2}{5} m R^2$

Ring / Hollow cyc] $I = m R^2$

Hollow sp.] $I = \frac{2}{3} m R^2$

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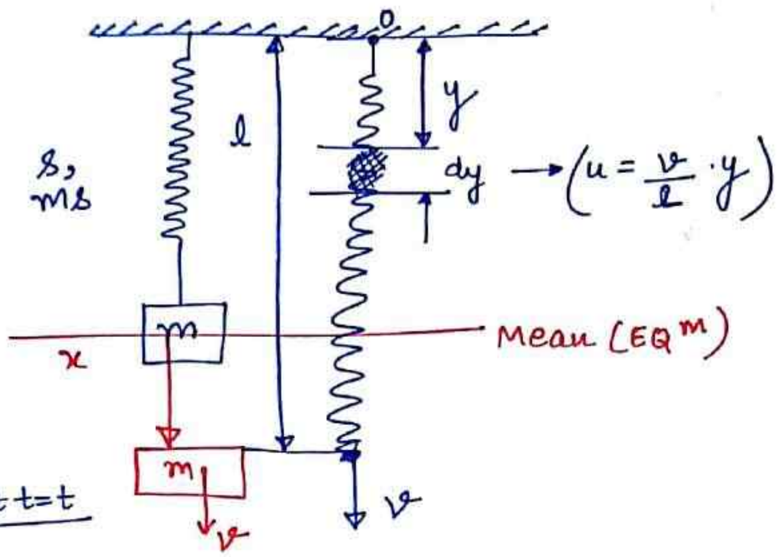
Spring-mass system
(spring is also having mass)



At t=t

$$KE_{spring} = \int_0^l \frac{1}{2}\left\{\frac{m_s}{l} \cdot dy\right\}\left\{\frac{v}{l} \cdot y\right\}^2$$

$$= \frac{1}{2} \cdot \frac{m_s}{\cancel{l}} \cdot \frac{v^2}{\cancel{l^2}} \cdot \frac{\cancel{l^3}}{3}$$

$$= \frac{1}{6} m_s v^2$$

At t=t

system

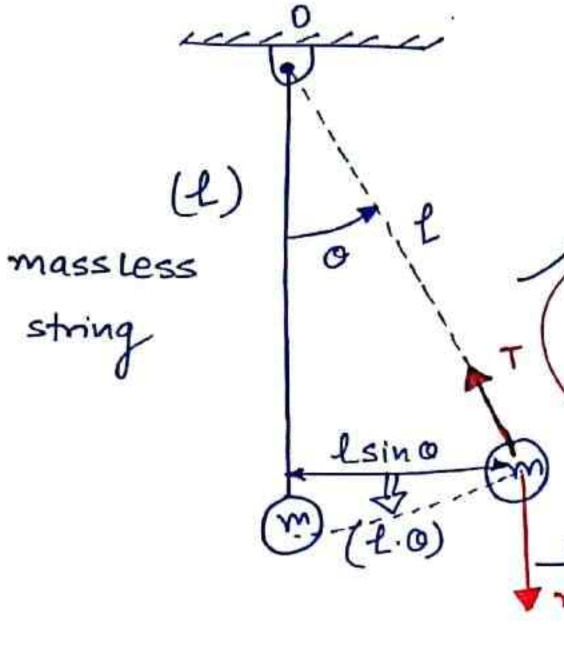
Energy

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{6} m_s v^2$$

$$= \frac{1}{2} s x^2 + \frac{1}{2}\left(m + \frac{m_s}{3}\right) v^2$$

$$\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}$$ **

Torque Method :- ** (very small oscillations)



θ

$(mg \cdot l \cdot \theta)$

$I\alpha = I\ddot{\theta}$ *

At t = t

$$I\ddot{\theta} + (mgl)\theta = 0$$

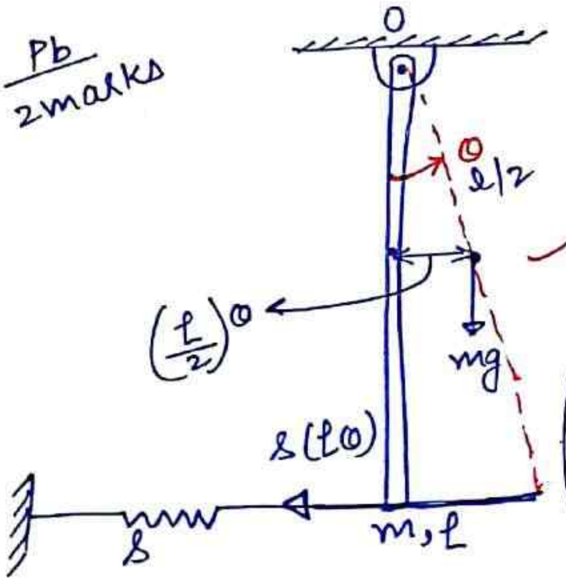
$$(\cancel{m}l^2)\ddot{\theta} + (\cancel{mgl})\theta = 0$$

$I = ml^2$

$$\ddot{\theta} + \left(\frac{g}{l}\right)\cdot\theta = 0 \quad **$$

$$\omega_n = \sqrt{g/l} \ \text{rad/s} \quad *$$

Pb
2marks



θ

$\left(mg\cdot\frac{l}{2}\cdot\theta\right)$

$(sl^2)\cdot\theta$

$I\ddot{\theta}$ *

$$I = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2$$

$$= \frac{m\ell^2}{12} + \frac{m\ell^2}{4}$$

$$I = \frac{m\ell^2}{3} \quad *$$

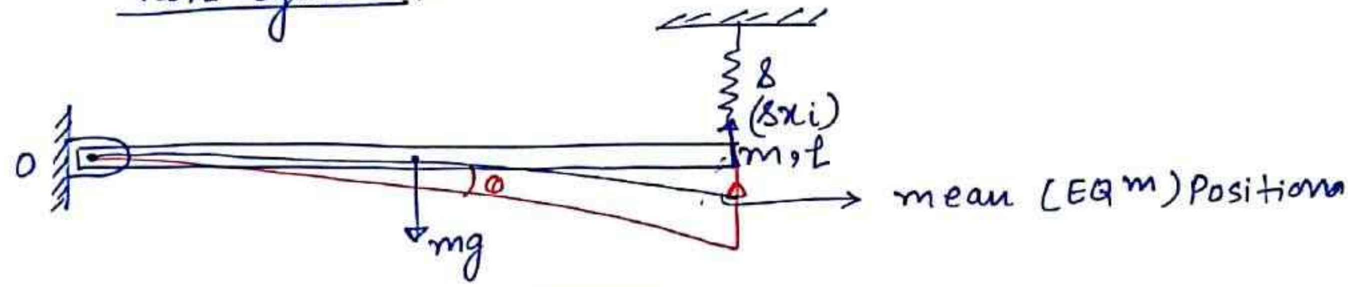
$$I\ddot{\theta} + \left[\left(\frac{mg\ell}{2}\right) + s\ell^2\right]\theta = 0$$

$$\ddot{\theta} + \frac{\left(\left(\frac{mg\ell}{2}\right) + sL^2\right)}{\left(\frac{m\ell^2}{3}\right)} \cdot \theta = 0 \quad *$$

$$\omega_n = \sqrt{\frac{\frac{mg\ell}{2} + s\ell^2}{\frac{m\ell^2}{3}}} \quad *$$

Note :-

Hori. system :-



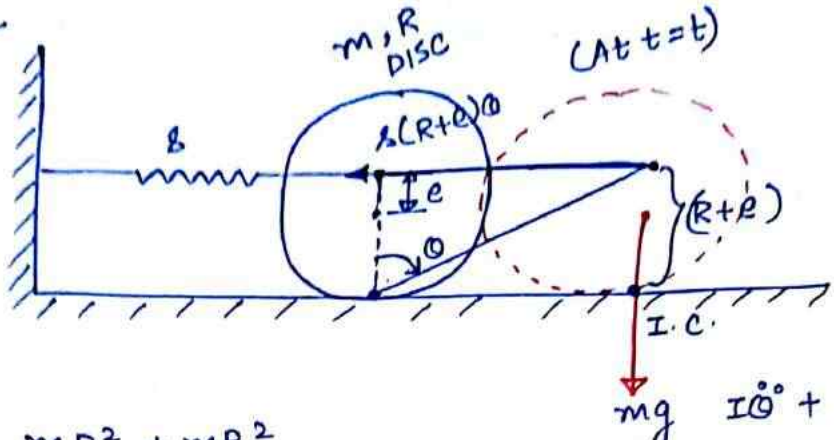
$$mg \cdot \frac{\ell}{2} = sx_i \times \ell \quad *$$

eq^m. eq^n.

mg Torque is balanced by sx_i Torque. Therefore mg torque will not be considered.

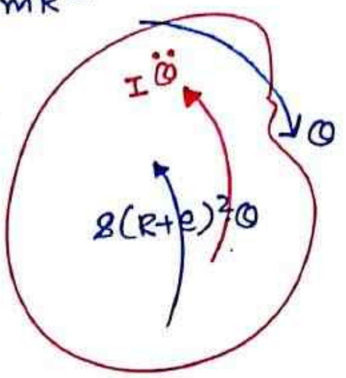
$$\omega_n = \sqrt{\frac{s\ell^2}{\frac{m\ell^2}{3}}} = \sqrt{\frac{3s}{m}} \text{ rad/s} \quad **$$

Pb
2 Marks



$$I\ddot{\theta} + k(R+e)^2 \cdot \theta = 0$$

$$I = \frac{mR^2}{2} + mR^2$$
$$= \frac{3}{2} mR^2$$

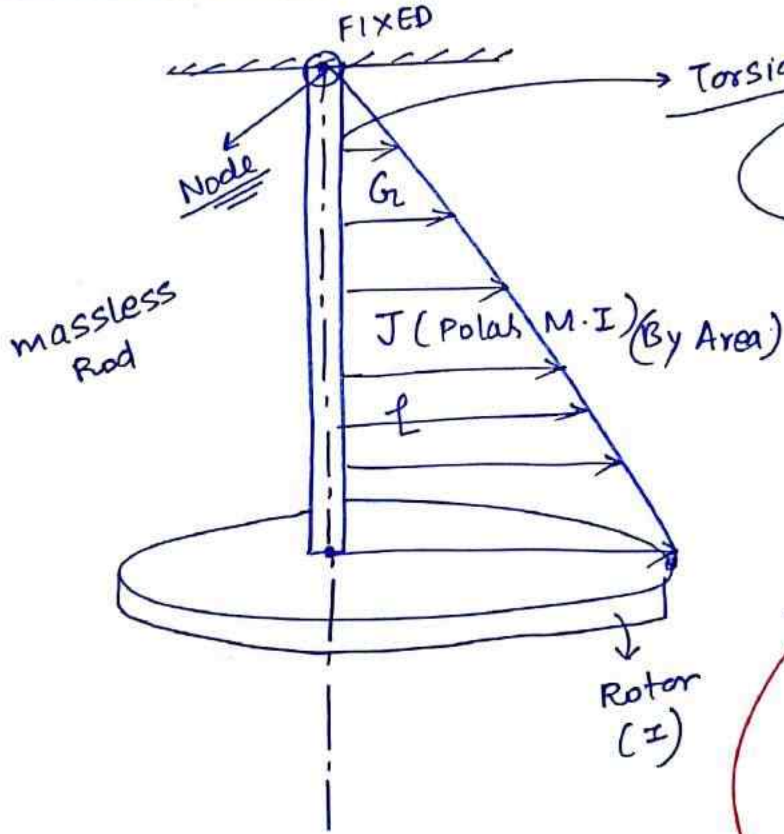


$$\frac{3}{2} mR^2 \cdot \ddot{\theta} + k(R+e)^2 \cdot \theta = 0$$

$$\ddot{\theta} + \frac{2k(R+e)^2}{3mR^2} \cdot \theta = 0$$ *

$$\omega_n = \sqrt{\frac{2k(R+e)^2}{3mR^2}}$$ **

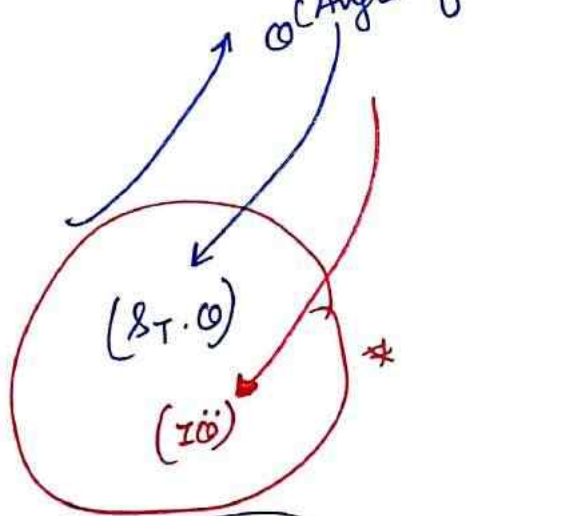
Torsional vibration :-



Torsional stiffness

$$k_T = \frac{GJ}{l}$$ *

θ (Angle of twist)



*

$$I\ddot{\theta} + (k_T) \cdot \theta = 0$$

$$\ddot{\theta} + \left(\frac{k_T}{I}\right) \cdot \theta = 0$$ *

$$\omega_n = \sqrt{\frac{k_T}{I}}$$ **

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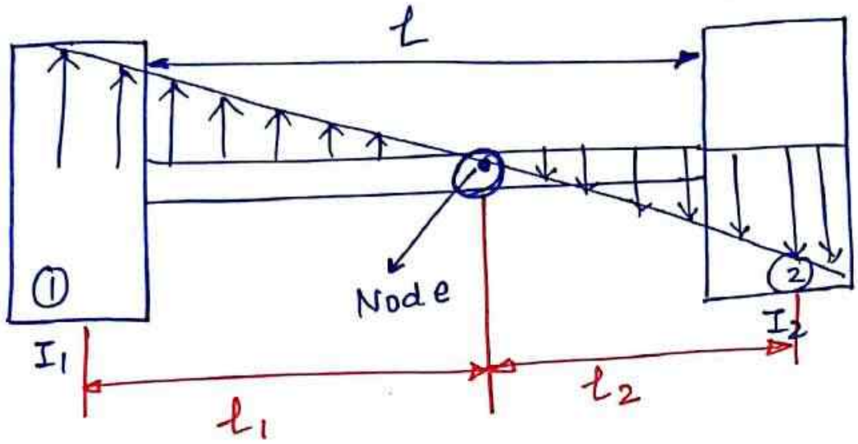
Note :- If Rod is having mass (I_{Rod})

Then :-

$$\boxed{\omega_n = \sqrt{\frac{\delta_T}{I + \frac{I_{Rod}}{3}}}} \quad **$$

If no. of Rotors = n
No. of Node point = (n-1)

Two Rotor System



$$l_1 + l_2 = l \quad \text{--- (1)}$$

$l_1 = ?$
$l_2 = ?$

$$\sqrt{\frac{\delta_{T_1}}{I_1}} = \sqrt{\frac{\delta_{T_2}}{I_2}}$$

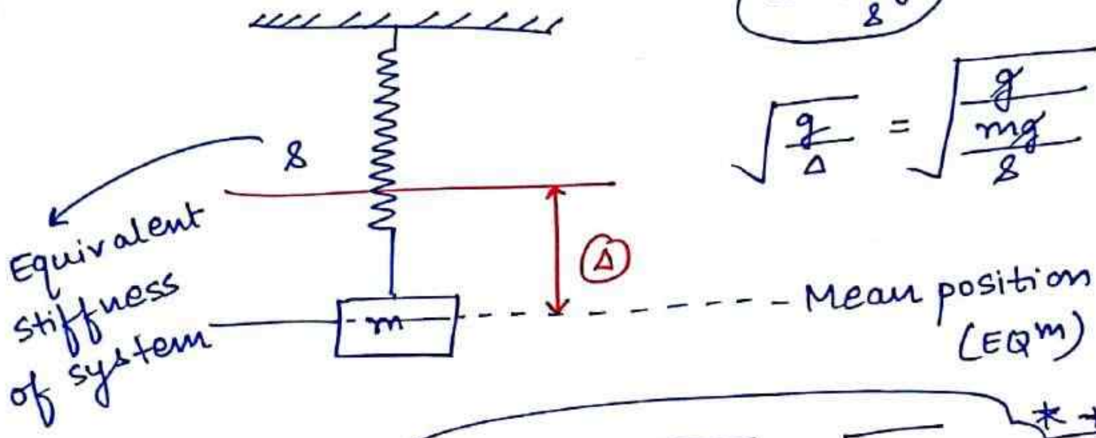
$$\frac{\delta_{T_1}}{I_1} = \frac{\delta_{T_2}}{I_2}$$

$$\frac{\cancel{GJ}}{l_1 I_1} = \frac{\cancel{GJ}}{l_2 I_2} \Rightarrow \frac{l_1}{l_2} = \frac{I_2}{I_1} \quad \text{--- (2)}$$

Rayleigh's Method :-

(Method of static Deflection of mass) (Δ)

1. Only applied in Natulal Vib.
2. Only applied in spring- mass systems.
3. There should be only one mass in the system and that must be a Point mass.

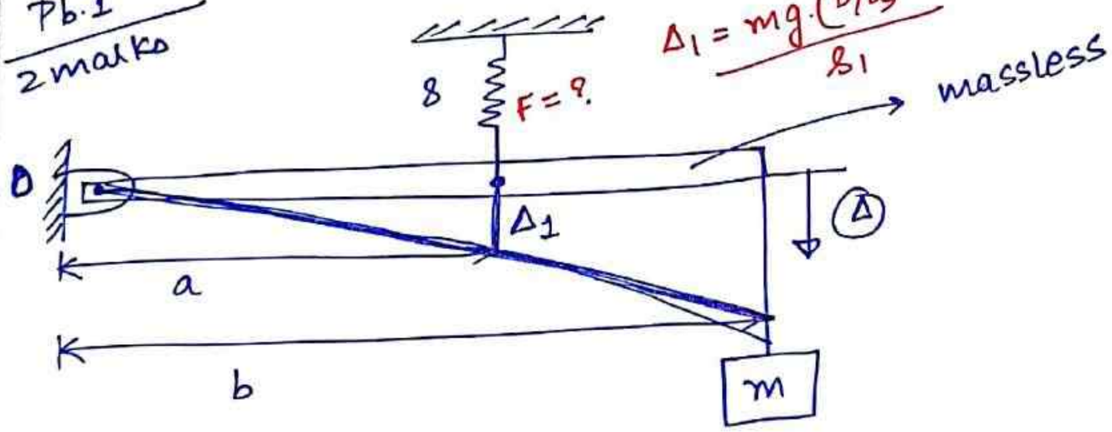


$$\Delta = \frac{mg}{8} *$$

$$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{\frac{mg}{8}}} = \sqrt{\frac{8}{m}} = \omega_n$$

$$\omega_n = \sqrt{\frac{8}{m}} = \sqrt{\frac{g}{\Delta}} \quad **$$

Pb. 1
2 marks



$$\Delta_1 = \frac{mg \cdot (b/a)}{8_1}$$

$$mg \cdot b = F \cdot a$$

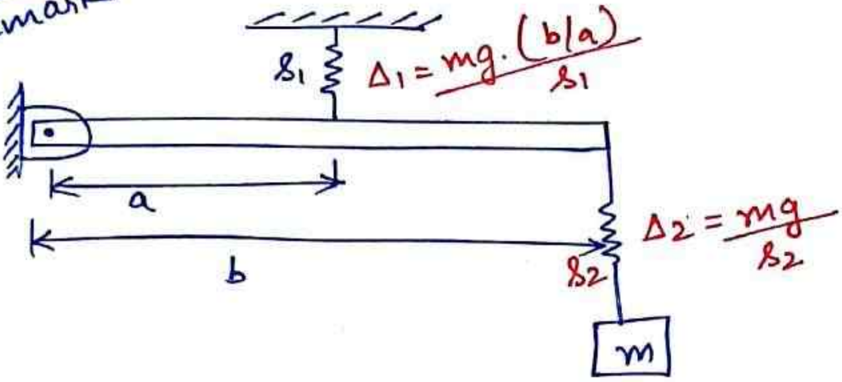
$$F = \left(mg \cdot b/a\right)$$

$$\frac{\Delta}{b} = \frac{\Delta_1}{a} \Rightarrow \Delta = \left(\Delta_1 \cdot \frac{b}{a}\right)$$

$$\Delta = \frac{mg}{s_1}\left(b/a\right)^2 \quad *$$

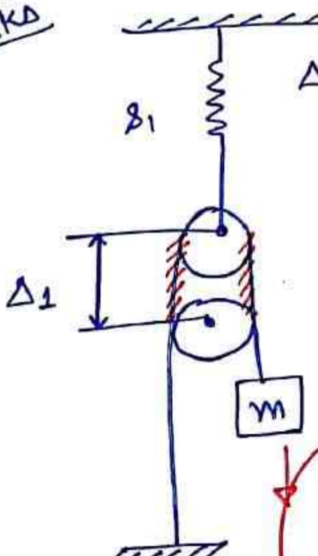
$$\omega_n = \sqrt{\frac{g}{\Delta}} = ?? \qquad = \sqrt{\frac{s}{m}} \rightarrow ??$$

Pb. 2
2 marks



$$\Delta = \left(\Delta_1 \cdot b/a + \Delta/_2\right) \quad *$$

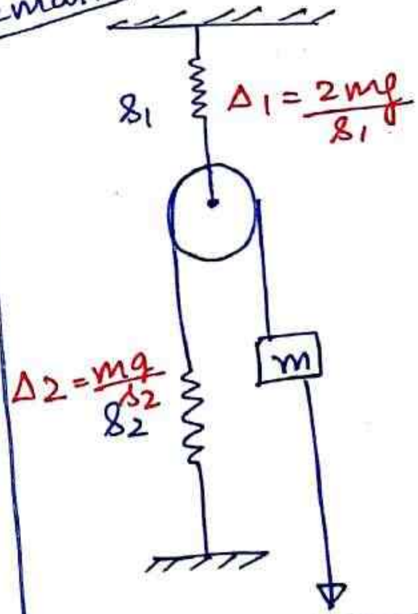
Pb 3
2 marks



$$\Delta_1 = \frac{2mg}{s_1}$$

$$\Delta = 2\Delta_1 = \frac{4mg}{s_1} \quad *$$

Pb 4
2 marks



$$\Delta = 2\Delta_1 + \Delta_2 \quad *$$

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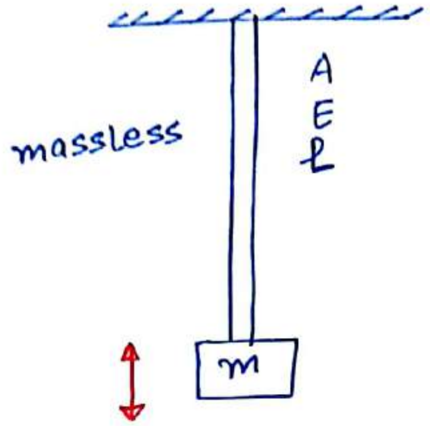
Longitudinal vib. of Beams :-

along the length.

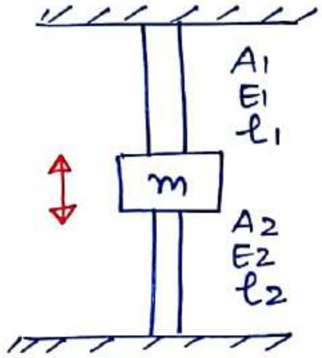


Longitudinal stiffness

$$s = \frac{AE}{l}$$

$$\omega_n = \sqrt{\frac{s}{m}} \rightarrow ?$$

*



$$s_1 = \frac{A_1 E_1}{l_1}$$

$$s_2 = \frac{A_2 E_2}{l_2}$$

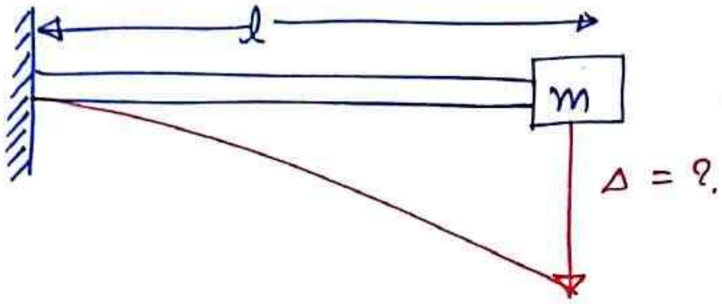
$$s = (s_1 + s_2)$$

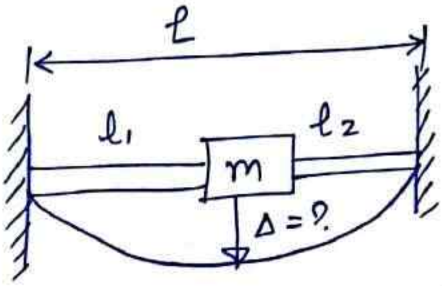
$$\omega_n = \sqrt{\frac{s}{m}}$$

*

Transverse Vib. of Beams :-

Vibr. in the dirn. of ⊥ar to length





$$\Delta = \frac{W l^3}{3EI} \ *$$

$W = mg$

I = Area M.I. of Beam about Bending Axis.

$$\Delta = \frac{W l_1^3 l_2^3}{3EI l^3} \ *$$

where

$W = mg$

$l = l_1 + l_2$

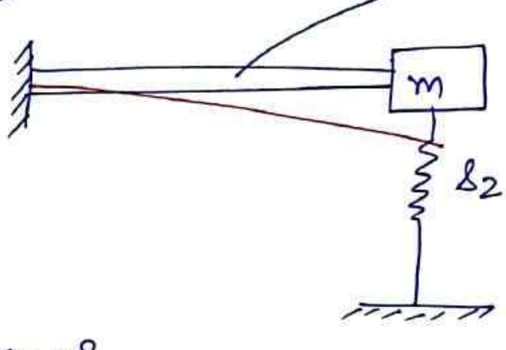
I

$$\omega_n = \sqrt{\frac{g}{\Delta}} = ?? \ * \qquad = \sqrt{\frac{s}{m}} \rightarrow ??$$

(Bending stiffness)

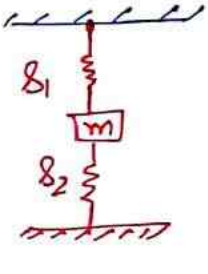
Pb
2marks

Bending stiffness ⇒ s_1



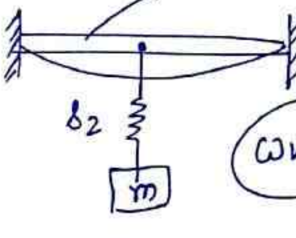
$\omega_n = ?$

|||



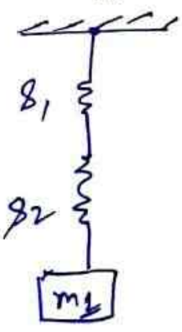
Pb
2marks

Bending stiffness ⇒ s_1

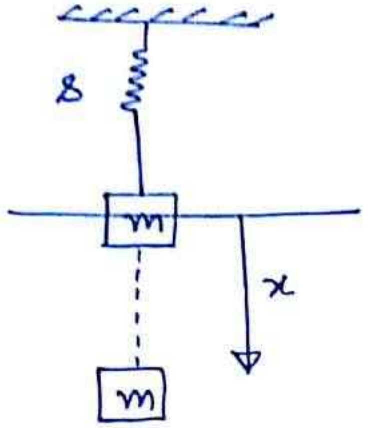


$\omega_n = ?$ *

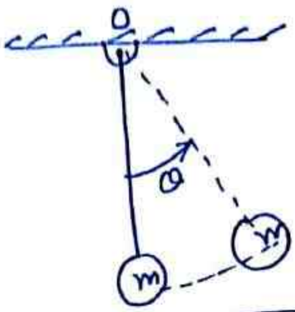
|||



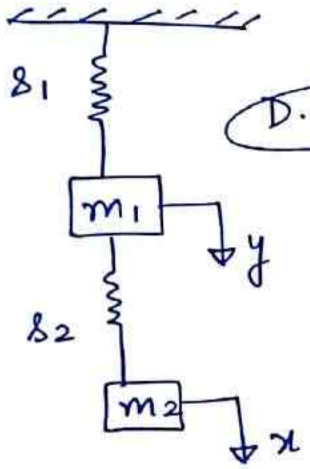
D.O.F. of a vibrating system :-



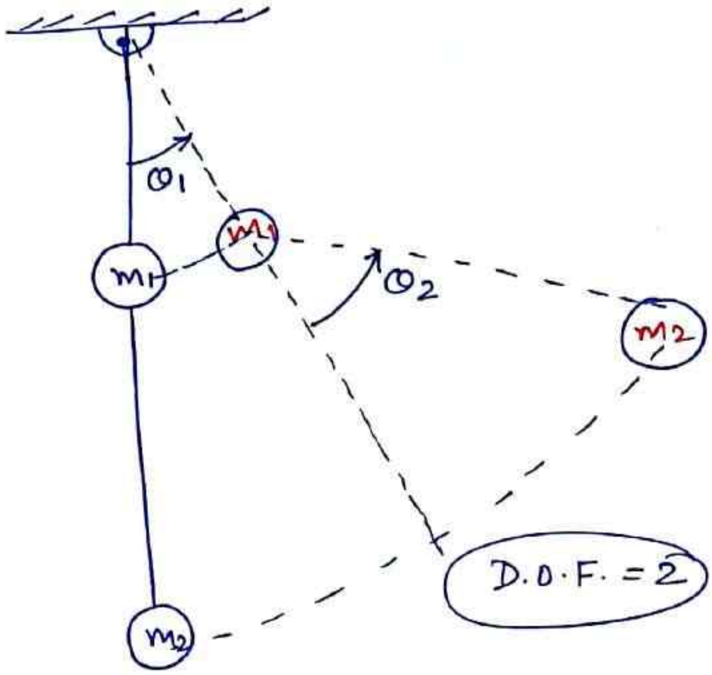
D.O.F. = 1 *



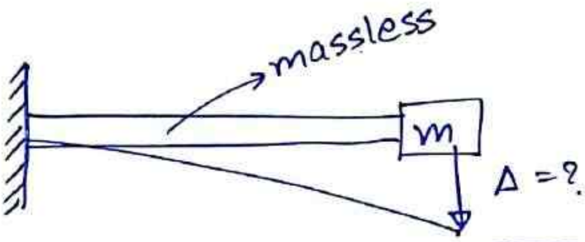
D.O.F. = 1 *



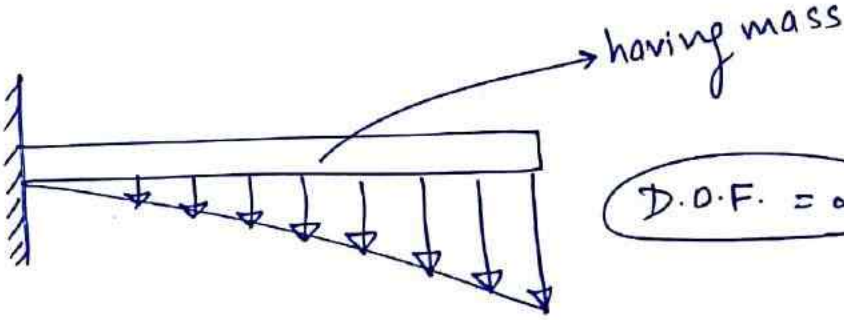
D.O.F. = 2 **



D.O.F. = 2 *



D.O.F. = 1 *



D.O.F. = ∞ **

Damped systems: (Kinetic friction $\neq 0$)

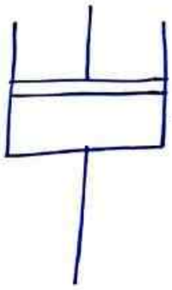
⇓

Technical name of kinetic friction in any ~~sub~~ vib. system

⇓

Damping and it is represented by the symbol

⇓



Damper

Damping in any system → Friction b/w dry surfaces. (coulamb damping) (very high)

Damping in any system → Viscous Damping (very less)

Damping force $\propto \dot{x}$

$= c\dot{x}$

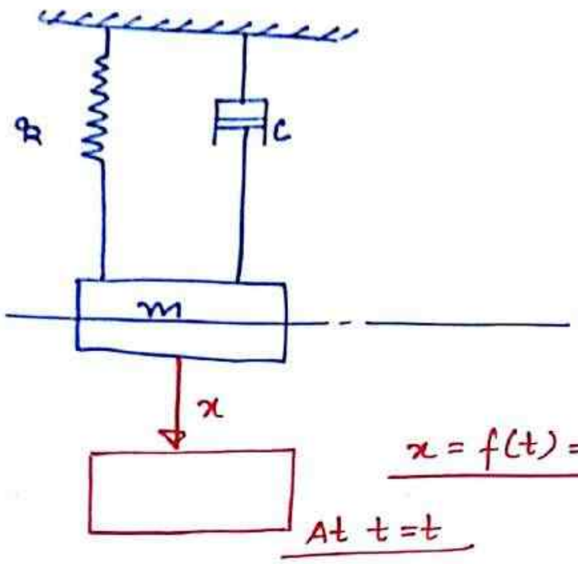
c → const. for a system (coeffi. of damping)

vibrn. → H.W. → 1 2 6 7 8 12 25 28 30 34 37 38 39 45 46 47 50 51 conven → T1 & T2

ans. is given in Hz

$\omega \to \frac{\div}{2\pi}$ by and check.

08/012/2016



Mean Position (EQm.) Position.

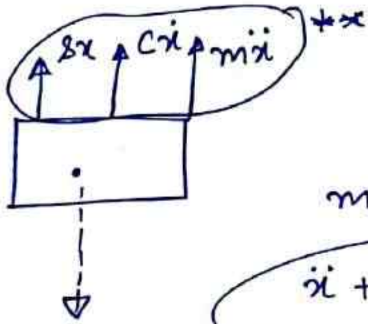
$x = f(t) = ??$

At t = t

At t = t

system

(F.B.D.)



**

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + (\omega_n^2)x = 0 \quad *$$

⇓

Eqn. of Damped system.

$\underbrace{\text{The soln. of the above eqn. is}}$

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad (\alpha_1 \neq \alpha_2)$$

OR *

$$x = (A + Bt)e^{\alpha t} \quad (\alpha_1 = \alpha_2 = \alpha)$$

A, B ⇓ const.

where :-

$\alpha_{1,2} \rightarrow$ Roots of auxiliary eqn.

⇑

$$\alpha^2 + (c/m)\alpha + (\omega_n^2) = 0$$

Scanned by CamScanner

where :-

$$\alpha_{1,2} = \frac{-c/m \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}}{2}$$

$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}$$

$$\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2} \Rightarrow \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2}} = \Rightarrow \text{Damping factor OR Damping Ratio.}$$

$$\zeta = \sqrt{\frac{\frac{c^2}{4m^{\cancel{2}}}}{\cancel{s/m}}} = \frac{c}{2\sqrt{sm}} \quad **$$

$$2\zeta\omega_n \; ****$$

$$\Downarrow$$

$$\cancel{2} \times \frac{c}{\cancel{2}\sqrt{\cancel{s}m}} \times \sqrt{\frac{\cancel{s}}{m}}$$

$$\Rightarrow (c/m)$$

Finally eqn. of Damped system

$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = 0 \quad **$$

The soln. will be :-

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad (\alpha_1 \neq \alpha_2)$$

OR *

$$x = (A + Bt)\,e^{\alpha t} \quad (\alpha_1 = \alpha_2 = \alpha)$$

A, B ⇓ const.

where :-

$$\alpha_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n \quad **$$

If $\underline{\zeta > 1}$ ⇒ overdamped systems
(over damping)
(coulamb damping)

If $\underline{\underline{\zeta = 1}}$

⇒ critically damped systems (critical damping)

If $\underline{\underline{\zeta < 1}}$

⇒ under damped systems (under damping)
(viscous damping)

1. Overdamped system
($\zeta > 1$)

The soln. will be

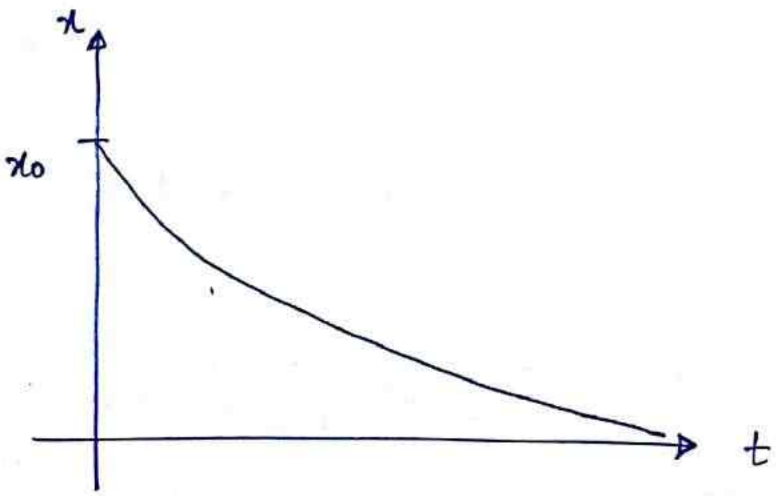
$$x = A e^{\alpha_1 t} + B e^{\alpha_2 t}$$

$$\boxed{x = A e^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + B e^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t}}$$ **

C.F.

No vibr. **

A, B const.



2. Critically damped system :-

$(\zeta = 1)$

$\alpha_1 = \alpha_2 = \alpha = -\omega_n$

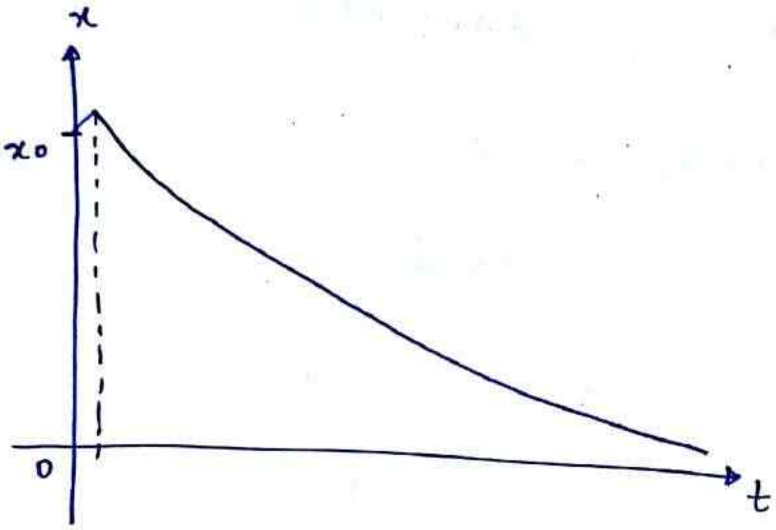
The soln. will be :-

$$x = (A + Bt)e^{-\omega_n t} *$$

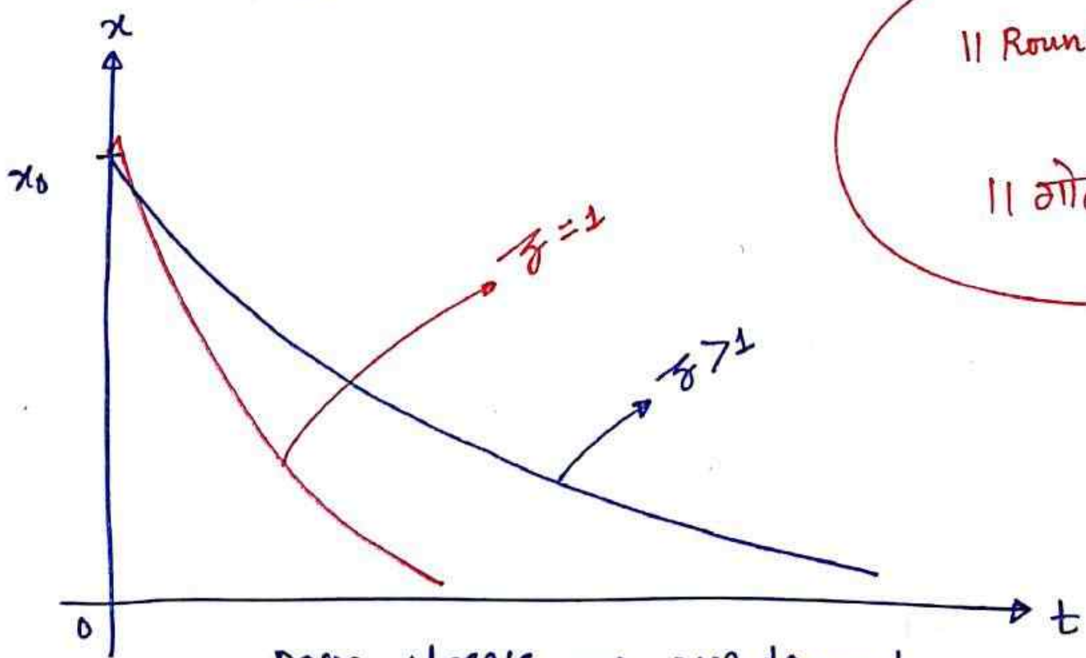
CF

[A, B] const.

No vib.



Note :- critical damping response is much fast than over damping.



AK-47

11 Rounds / sec

11 गोली → एक sec

Door closers → overdamped

(AK-47) → critically damped

rounds

660/min

3. Underdamped systems :-
$(\zeta < 1)$

$$\alpha_{1,2} = -\zeta\omega_n \pm i\underbrace{\sqrt{1-\zeta^2}\,.\omega_n}_{\omega_d = \text{const.}}$$

$$\boxed{\alpha_{1,2} = (-\zeta\omega_n \pm i\omega_d)} \;**$$

The soln. will be :-

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = Ae^{(-\zeta\omega_n + i\omega_d)t} + Be^{(-\zeta\omega_n - i\omega_d)t}.$$

$$x = e^{-\zeta\omega_n t}\left[\underbrace{(A+B)}_{X\sin\phi}\cos\omega_d t + \underbrace{i(A-B)}_{X\cos\phi}\sin\omega_d t\right]$$

$$\boxed{x = Xe^{-\zeta\omega_n t}.\sin(\omega_d t + \phi)} \;** \quad \underbrace{X, \phi}_{\Downarrow\ \text{const.}}$$

$$x = Xe^{-\zeta\omega_n t}.\sin(\omega_d t + \phi)$$

CE

Amplitude
$\Downarrow$
$\downarrow\downarrow$ fn(time)

VIB. with freq.

$$\omega_d = \sqrt{1-\zeta^2}\,.\omega_n$$

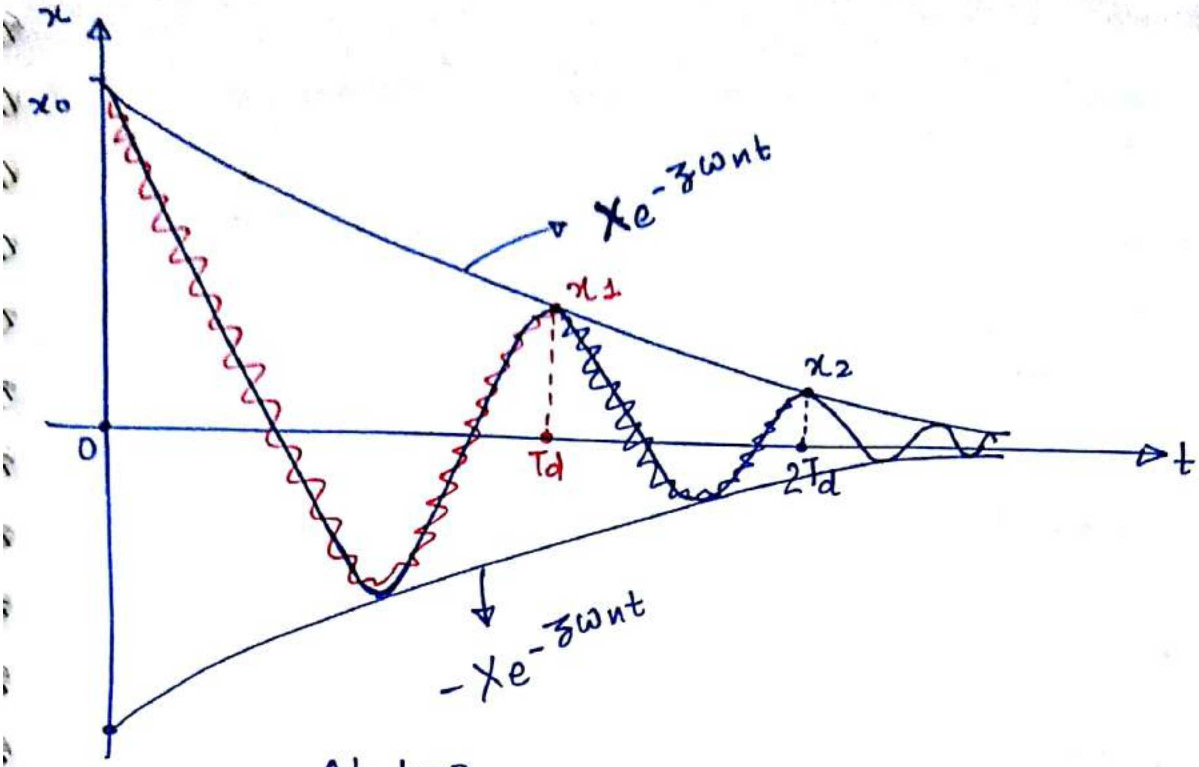
$\Rightarrow$ const.

$$T_d = \frac{2\pi}{\omega_d} \;(\text{sec})$$

$\Rightarrow$ const

$$f_d = \frac{\omega_d}{2\pi} \;(Hz)$$

$\Rightarrow$ const.



At $t=0$

$$x_0 = X\sin\phi$$

At $t = T_d$

$$x_1 = Xe^{-\zeta\omega_n(T_d)} \cdot \sin\phi$$

At $t = 2T_d$:-

$$x_2 = Xe^{-\zeta\omega_n(2T_d)} \cdot \sin\phi$$

Decrement Ratio :-

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \text{-----------} = e^{\zeta \omega_n T_d} = \text{const.}$$
$$= e^{\delta}.$$

**

logarithmic Decrement :-
(δ)

$$\delta = \ln\left(e^{\zeta \omega_n T_d}\right)$$

$$= \zeta \omega_n T_d$$

$$= \frac{\zeta \cdot \cancel{\omega_n} \cdot 2\pi}{\sqrt{1-\zeta^2} \cdot \cancel{\omega_n}}$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$ **

Critical damping coefficient :-
(C_c) :

$$\cancel{2}\zeta \cancel{\omega_n} = c/\cancel{m}$$

$$\cancel{2} \times 1 \times \cancel{\omega_n} = \frac{C_c}{\cancel{m}}$$

$$\zeta = c/C_c$$

**

$$\zeta = \frac{\text{Actual damping coeff.}}{\text{Critical} \;\text{---}\,''\,\text{---}}$$ **

Note :- **

- The Ratio of displacements of end of 3rd cycle to the start of 8th cycle is 2.5

$$\frac{x_3}{x_7} = 2.5$$ **

- The Ratio of displacements of 3rd cy. to 8th cy. is 2.5.

$$\frac{x_3}{x_8} = 2.5$$ **

Pb - 55
15 Marks
IES

$m = 7.5\,kg$

$T_d = \frac{35}{60}$ sec

$\omega_d = \frac{2\pi}{T_d} \Rightarrow$ known

$\frac{x_1}{x_7} = 2.5$

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n \rightarrow$??

(i) $\omega_n = \sqrt{\frac{s}{m}} \rightarrow$??

(ii) $2\zeta\omega_n = \frac{c}{m} \rightarrow$??

$\zeta = \frac{c}{C_c} \rightarrow$??

$$\frac{x_1}{x_2}\cdot\frac{x_2}{x_3}\cdot\frac{x_3}{x_4}\cdot\frac{x_4}{x_5}\cdot\frac{x_5}{x_6}\cdot\frac{x_6}{x_7} = 2.5$$

$$e^{6\delta} = 2.5$$

$$6\delta = \ln 2.5$$

$$\frac{6 \times 2\pi\zeta}{\sqrt{1-\zeta^2}} = \ln 2.5$$

$\zeta = ?$ *

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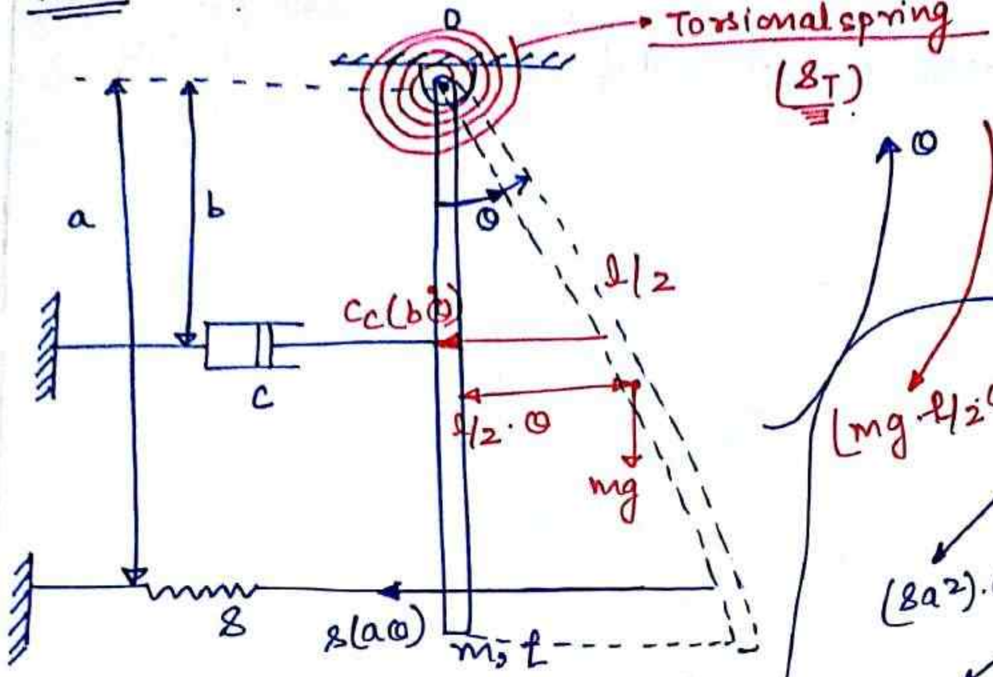
Note :-



θ

$\left(mg \frac{l}{2} \theta\right)$

$(sa^2)\cdot\theta$

$(s_T \cdot \theta)$

$(cb^2)\dot{\theta}$

$I\ddot{\theta}$

**

$$\boxed{I = \frac{ml^2}{3}} \quad **$$

$$I\ddot{\theta} + (cb^2)\dot{\theta} + \left(\frac{mgl}{2} + sa^2 + s_T\right)\theta = 0 \quad **$$

**

⇓

Eqn. of Damped system

$$(m\ddot{x} + c\dot{x} + sx) = 0$$

Pb1
2 marks

The damp. coeff. in the vibrn. eqn. will be :-

$C b^2$ Ans

Pb 2
2 marks

$\omega_n = ?$ *

$$\omega_n = \sqrt{\frac{\frac{mgl}{2} + Sa^2 + S_T}{I}}$$ **

Pb 3
2 marks

$\zeta = ?$

$$2\zeta\omega_n = \frac{Cb^2}{I}$$

??

Pb 4
2 marks

$C_c = ?$ *

$$2\omega_n = \frac{C_c b^2}{I}$$

??

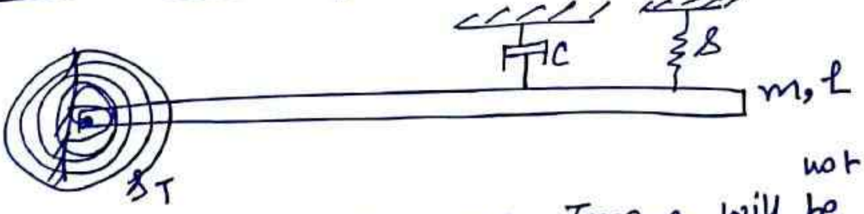
Pb 5
2 marks

$\omega_d = ?$ *

$$\omega_d = \sqrt{1-\zeta^2} \cdot \omega_n$$ *

??

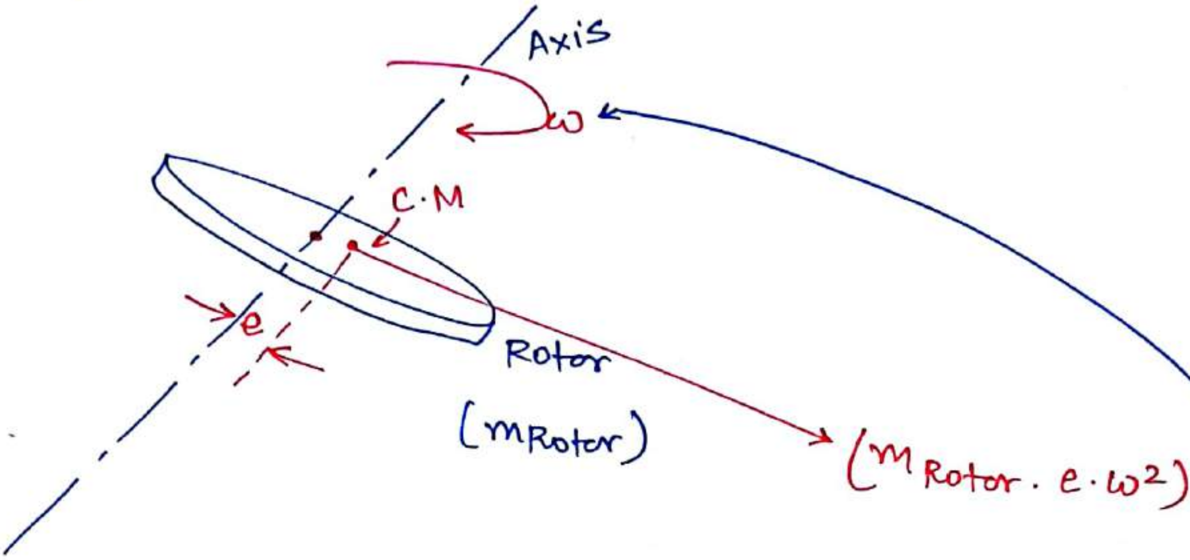
Note :- Hor. system



Here mg Torque will not be considered.

Vibration causing unbalanced forces in Running mechanical system :-

Rotating unbalance :-



At t = t ;

$$F_{UN} = (m_{Rotor} \cdot e \cdot \omega^2) \sin\theta$$

$$= (m_{Rotor} \cdot e \cdot \omega^2) \sin \omega t$$

* $$\boxed{F_{UN} = F_0 \sin \omega t}$$

$F_0 \rightarrow$ max. value

$\omega \rightarrow$ force freq. (excitation Freq.) (speed)

** Reci. unbalance :-

At $t = t$

$$F_{UN} = (m_{Reci} \cdot r \cdot \omega^2) \sin\theta$$

$$= (m_{Reci} \cdot r \cdot \omega^2) \sin\omega t$$

** $\boxed{F_{UN} = F_0 \sin\omega t}$

$F_0 \rightarrow$ max. value

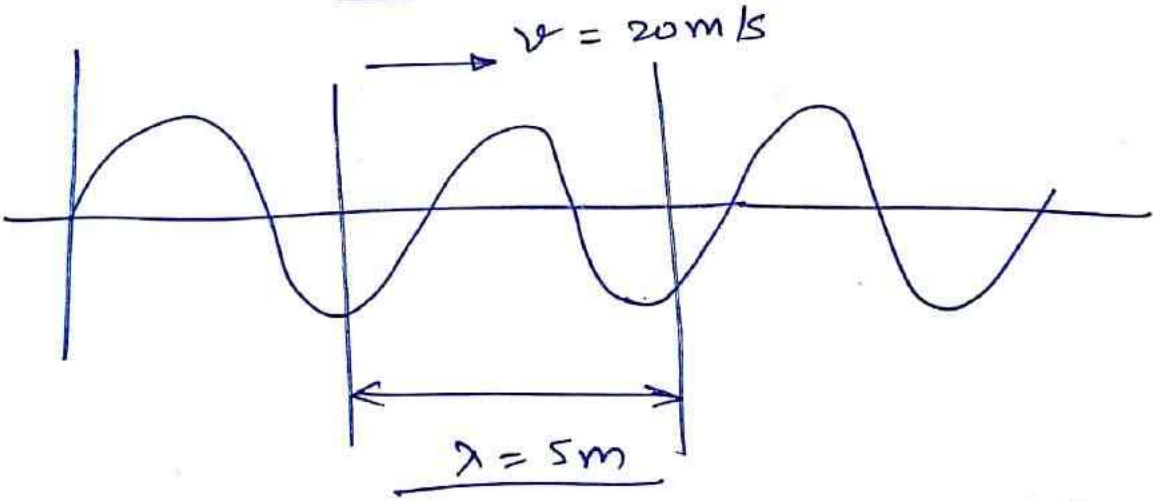
$\omega \rightarrow$ Force Frequency (speed)

$r \rightarrow$ crank Radius

$\Rightarrow \frac{(\text{stroke})}{2}$

wave form

$F_0 \longrightarrow$ 200 N



$$V = f\lambda$$

$$20 = f \times 5$$

$$f = 4\ Hz$$

$$\omega = 2\pi \times 4 = 8\pi\ rad/s$$ *

Direct form :-

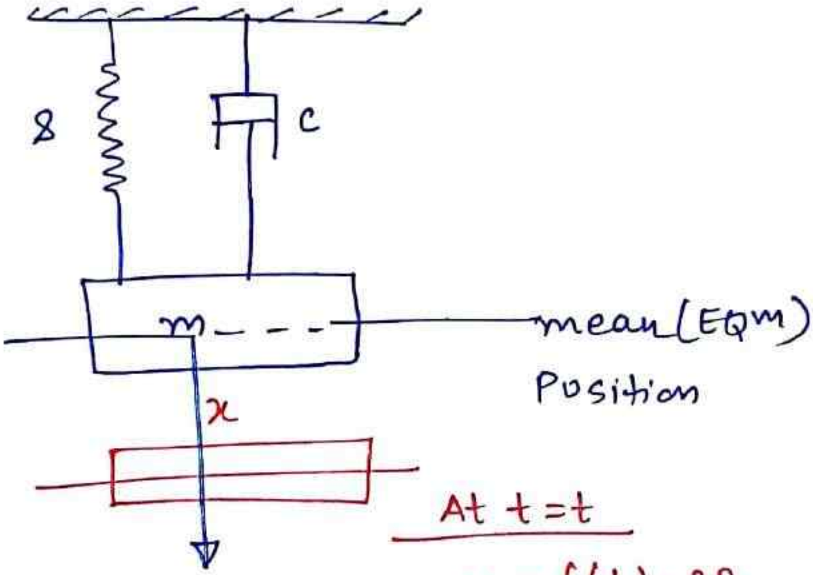
FUN = 200 cos 3t

200 → F_o *

3 → ω *

$\underline{\underline{m}}$ → machine mass which is under vibrations **

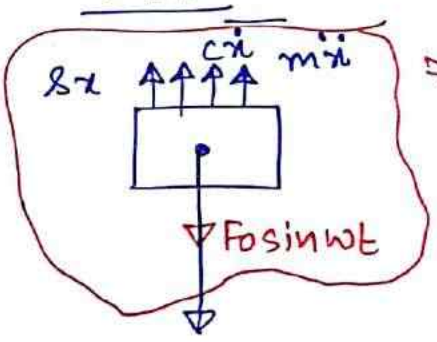
Forced-damped Systems :- (Running m/c analysis)



At t = t

$x = f(t) = ??$

At t=t system F.B.D.



**

FUN = Fo sin ωt

Fo → max^m. value

ω → Force freq. (speed)

$$m\ddot{x} + c\dot{x} + sx - F_o \sin\omega t = 0$$

$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = \left(\frac{F_o}{m}\right)\sin\omega t$$ *

eq^n. of forced-damped system.

The sol^n. will be

$$x = CF + PI$$ **

$\underline{CF}$:-

$\begin{cases} \zeta > 1 \\ \zeta = 1 \end{cases}$ No vib.

$\zeta < 1$] vib.

after sometime $CF \rightarrow 0$.

$\underline{\text{After sometime}}$

$\hookrightarrow CF \rightarrow 0$

$x = PI$ **

$\underline{PI}$:-

($D \rightarrow$ operator)

$(D^2 = -\omega^2)$

$$PI = \frac{\frac{F_0}{m} \sin\omega t}{D^2 + (2\zeta\omega_n)D + \omega_n^2}$$

$$= \frac{\frac{F_0}{m} \sin\omega t}{(\omega_n^2 - \omega^2) + (2\zeta\omega_n)D} \cdot \frac{\{(\omega_n^2 - \omega^2) - (2\zeta\omega_n)D\}}{\{(\omega_n^2 - \omega^2) - (2\zeta\omega_n)D\}}$$

$$= \frac{\frac{F_0}{m}\left[\underbrace{(\omega_n^2 - \omega^2)}_{R\cos\phi} \cdot \sin\omega t - \underbrace{(2\zeta\omega\omega_n)}_{R\sin\phi} \cdot \cos\omega t\right]}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}$$

$$= \frac{F_0}{m} \cdot \frac{\cancel{R}\sin(\omega t - \phi)}{R^{\cancel{2}}}$$

$$= \frac{\left(\frac{F_0}{m}\right) \cdot \sin(\omega t - \phi)}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}}$$

$$PI = \frac{(F_0/s)}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}} \cdot \sin(\omega t - \phi)$$ **

VIB with Freq. (ω)

for all values of ξ

(After sometime)

$CF \to 0$

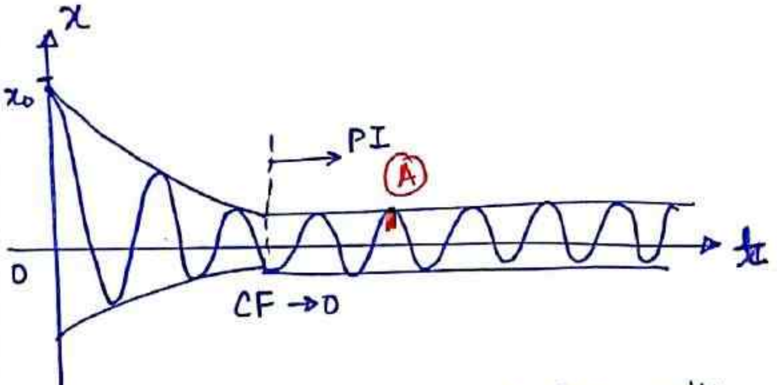
$x = PI$

$$\boxed{x = A\sin(\omega t - \phi)} *$$

$$A = \frac{(F_0/s)}{\sqrt{\left\{1-(\omega/\omega_n)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

Amplitude of steady state vib. (forced vib.)

Independent of time



VIB in Running system will never stop.

To stop the Fatigue, every mechanical Running system must have one Running life, Fatigue life

$$MF = \frac{A}{(F_0/s)}$$

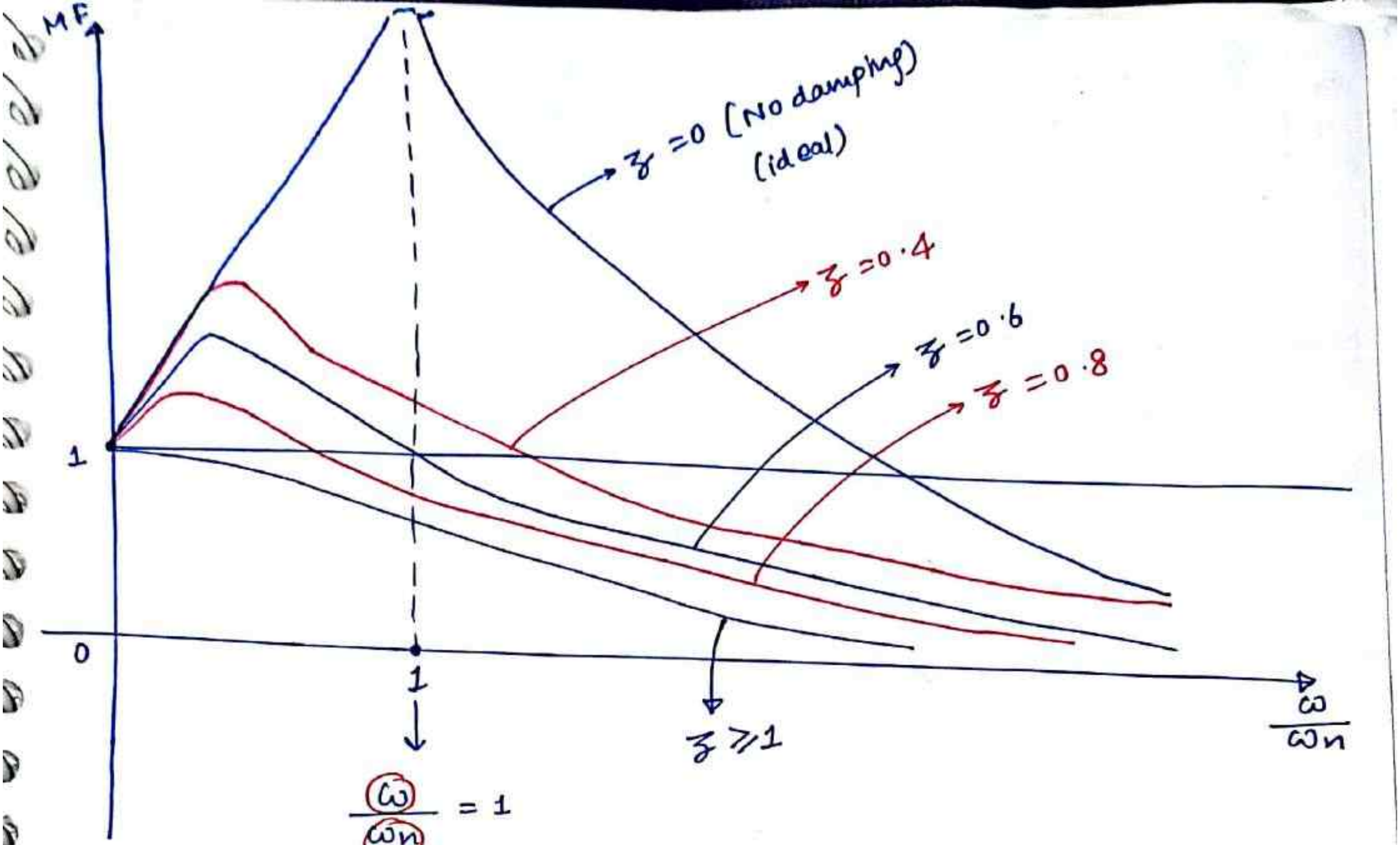
magnification factor (strength of A)

$$MF = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left[\frac{2\xi\omega}{\omega_n}\right]^2}}$$ **

depends upon

(i) $\frac{\omega}{\omega_n}$

(ii) ξ **



$\frac{\omega}{\omega_n} = 1$

(Resonance)

1. ↑ underdamping

⇒ ζ ↓

⇒ MF ↑ ⇒ A ↑

⇒ (Running life ↓↓

2. Amax at (MFmax at)

at $\frac{\omega}{\omega_n} < 1$ (under damping)

(viscous ")

at $\frac{\omega}{\omega_n} = 1$ (No damping)

at $\frac{\omega}{\omega_n} = 0$ (over /critical damping)

(coulomb)

3. (A) resonance $\left(\frac{\omega}{\omega_n}=1\right)$ $= \frac{(F_0/s)}{2\zeta} \propto \frac{1}{\zeta}$ **

**

Note :- After sometime

$$x = A\sin(\omega t-\phi)$$

$$\dot{x} = A\omega\cos(\omega t-\phi)$$

$$= A\omega \sin\left[\frac{\pi}{2}+(\omega t-\phi)\right]$$

$$\ddot{x} = -A\omega^2 \sin(\omega t-\phi)$$

Basic eqn. was :-

$$F_0\sin\omega t - m\ddot{x} - c\dot{x} - sx = 0$$

$$\underbrace{F_0}_{F_{un}\,(max.)}\sin\omega t + \underbrace{(m\omega^2 A)}_{F_I\,(max.)}\sin(\omega t-\phi) - \underbrace{(C\omega A)}_{F_{damping}\,(max.)}\sin\left[\frac{\pi}{2}+(\omega t-\phi)\right]$$

$$- \underbrace{(sA)}_{F_{spring}\,(F_{elastic})\,(max.)}\sin(\omega t-\phi) = 0$$

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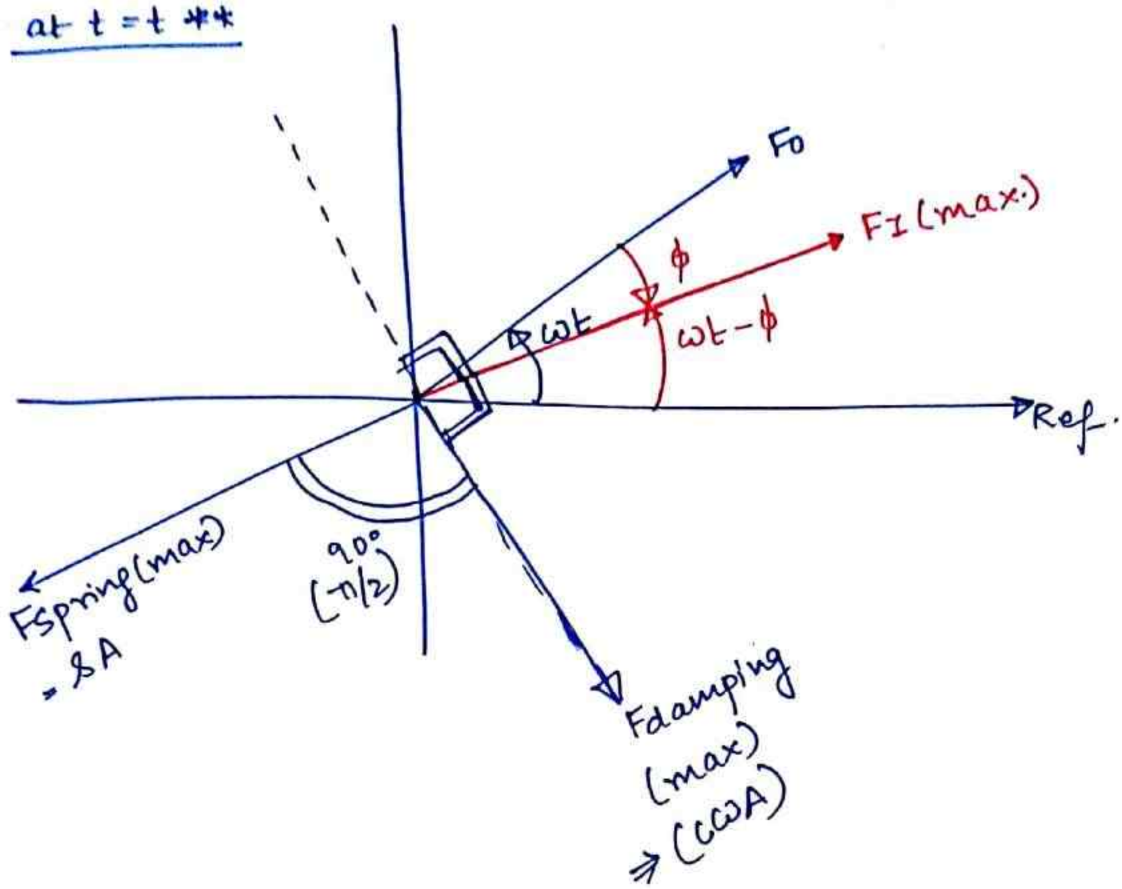
Phasor :-

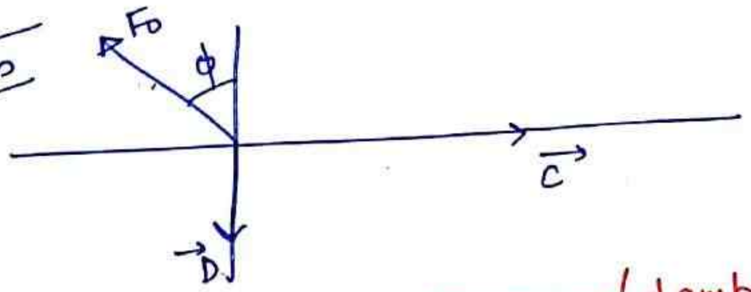
at t = t **



" At any moment, the max. Responses of spring and damping forces are mutually ⊥ar to each other. "

**

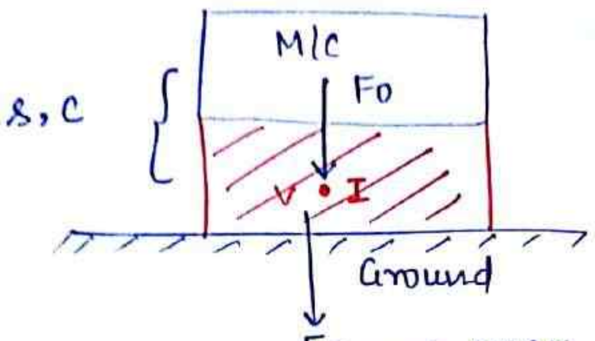
Pb
2marks



$\vec{C} \longrightarrow$? → (damping force) max

$\vec{D} \longrightarrow$? → (spring force) max.

VIBRATION ISOLATION :-
(Foundation)



$F_T \longrightarrow$ max. Transmitted here to ground

$\boxed{F_T <<<<<<< F_0}$

$\epsilon = \frac{F_T}{F_0}$

$\downarrow$

Transmissibility

(Performance of V.I system).

$0 < \epsilon < 1$ **

$\epsilon \longrightarrow 0$

(Best)

$$F_T = \sqrt{(sA)^2 + (c\omega A)^2}$$

$$F_T = sA\sqrt{1 + \left(\frac{c\omega \cancel{A}/m}{s\cancel{A}/m}\right)^2}$$

$$F_T = sA\sqrt{1 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}$$

$$F_0 = sA\sqrt{\left\{1 - \left(\omega/\omega_n\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}$$

$$\epsilon = \frac{F_T}{F_0}$$

$$\epsilon = \frac{\sqrt{1+\left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}$$

**

depends upon

(i) ω/ω_n

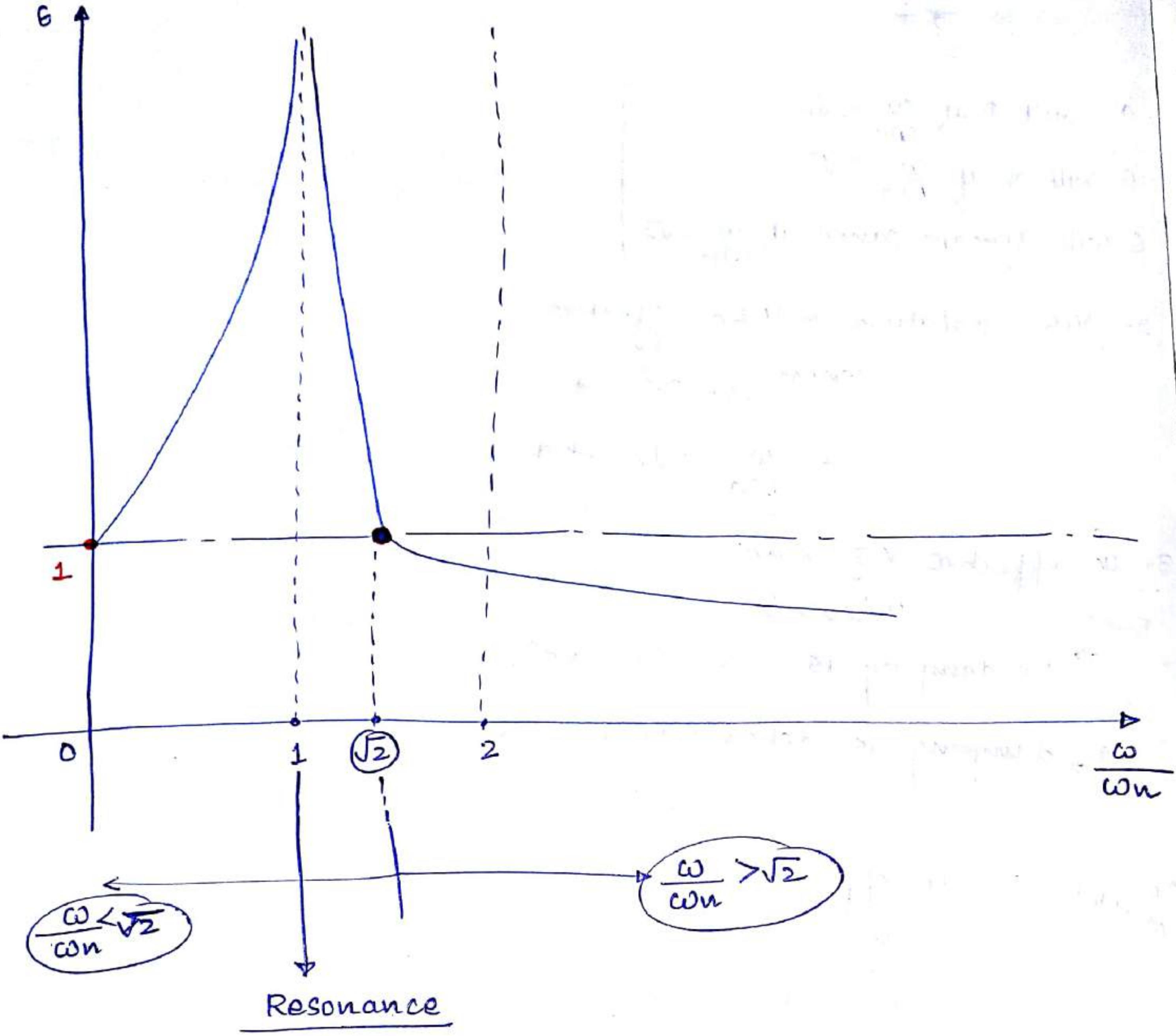
(ii) ζ

If $\frac{\omega}{\omega_n} = 0$, $\epsilon = 1$

If $\frac{\omega}{\omega_n} = \sqrt{2}$, $\epsilon = 1$

for all values of ζ

"N.P."



G
1
0
1
√2
2
ω/ωn
ω/ωn < √2
ω/ωn > √2
Resonance

1. $\uparrow$ underdamping

$\Rightarrow \underline{\underline{\xi \downarrow}}$

ϵ will $\uparrow$ if $\frac{\omega}{\omega_n} < \sqrt{2}$

ϵ will $\downarrow$ if $\frac{\omega}{\omega_n} > \sqrt{2}$

ϵ will remain same if $\frac{\omega}{\omega_n} = \sqrt{2}$

2. VIB. isolation will be effective

when $0 < \epsilon < 1$ *

$\Rightarrow \frac{\omega}{\omega_n} > \sqrt{2}$ **

3. In effective V.I. Zone,

Rubbers $\omega/\omega_n > \sqrt{2}$

$\Rightarrow$ No damping is Best $(\epsilon \to 0)$,

$\Rightarrow$ damping is detrimental (harmful),

Pb 2marks

For effective V.I.

$\omega_n = ?$ *

(i) ω

(ii) 2ω

(iii) $\omega/4$ ✓

(iv) 10ω

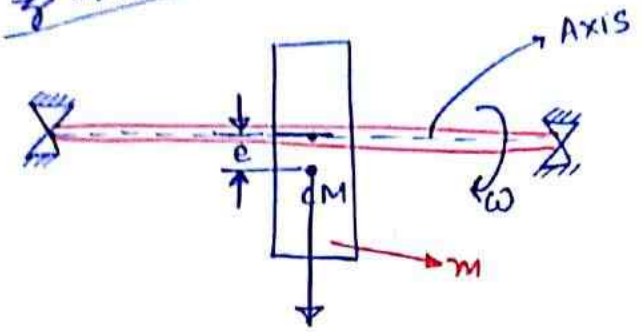
For effective V.I. :-

$\omega/\omega_n > \sqrt{2}$

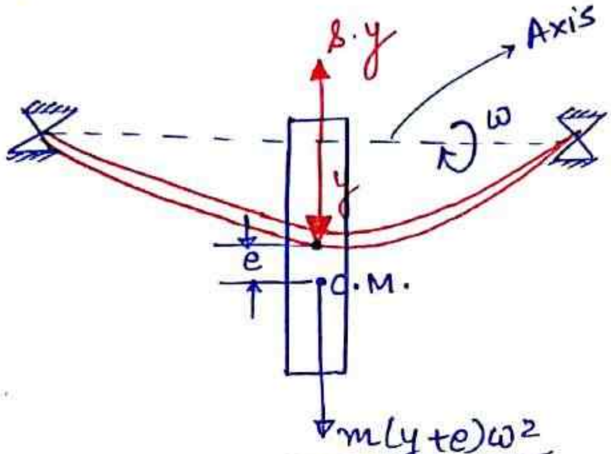
$\omega_n < \omega/\sqrt{2}$ *

Whirling of shafts :-

$\zeta \to 0$



After sometime



s → Bending stiffness

$$m(y+e)\omega^2 = s \cdot y$$

$$my\omega^2 + me\omega^2 = s \cdot y$$

$$\cancel{m}e\cancel{\omega^2} = \cancel{m}y\cancel{\omega^2}\left(\frac{s}{m\omega^2} - 1\right)$$

$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} \quad **$$

(A) $\zeta = 0$

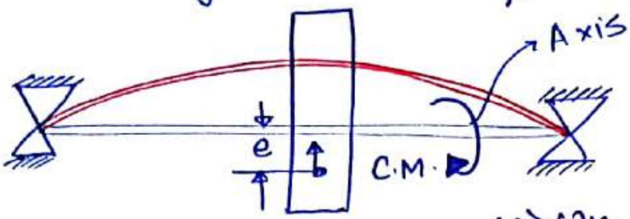
In some systems
Running life is very less

1. Either the vib. are too heavy
2. or the items are too delicate

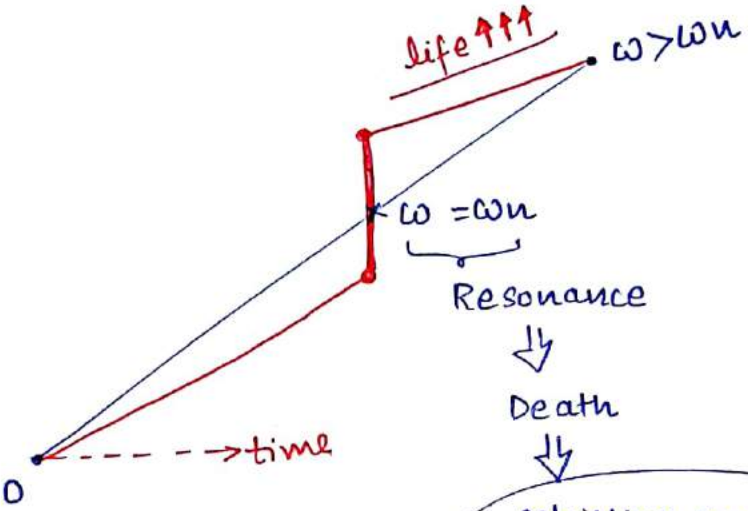
To ↑ the Running life

if $\omega > \omega_n$

⇒ shaft Bending will be in opposite direction



$\omega > \omega_n$



Resonance

⇓

Death

⇓

whirling speed

⇓

$\sqrt{\frac{s}{m}}$

*

Critical speed (Fundamental whirling speed)

Second critical speed:

$\Downarrow$

Due to self weight (mg)

$\Downarrow$

only in Horiz. shaft.

Second critical (whirling) speed

$$\approx \frac{1}{2}\omega_n$$

$$\approx \frac{1}{2}\sqrt{\frac{s}{m}} \text{ rad/s}$$

Pb. 56
IES
15 marks

$m = 17$ kg

$s = 1000$ N/m

$$\omega_n = \sqrt{\frac{1000}{17}} \text{ rad/s}$$

$\omega/\omega_n \longrightarrow$ KNOWN

$\zeta = 0.20$

$m_{Rec} = 2$ kg

$r = \frac{75}{\cancel{2}1000}$ meter

$N = 500$ rpm $\rightarrow \omega = \frac{2\pi \times 500}{60}$ rad/s

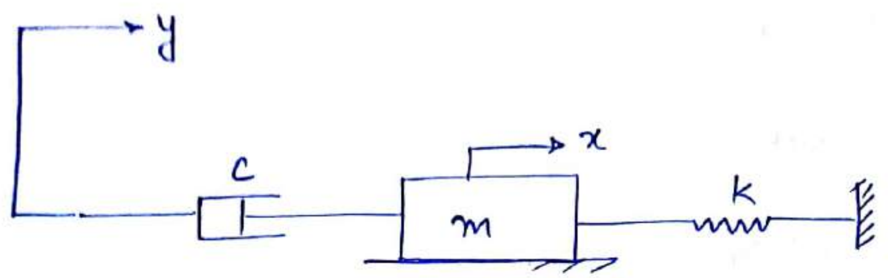
$$F_0 = 2 \times \frac{75}{2000} \times \left(\frac{2\pi \times 500}{60}\right)^2 \text{ N}$$

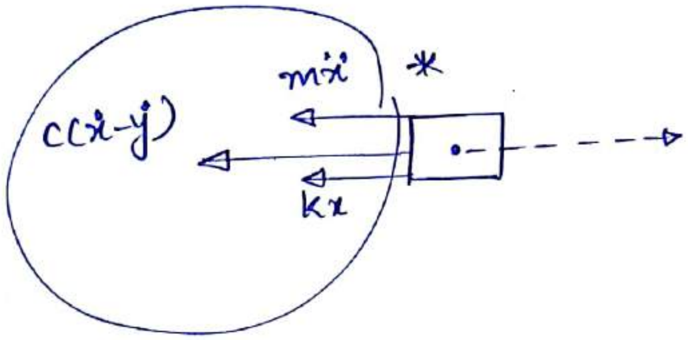
(i)
$$A = \frac{(F_0/s)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[\frac{2\zeta\omega}{\omega_n}\right]^2}} = ??$$

(ii)
$$\epsilon = \frac{\sqrt{1+\left[\frac{2\zeta\omega}{\omega_n}\right]^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[\frac{2\zeta\omega}{\omega_n}\right]^2}} = ?? = \frac{F_T}{F_0} \rightarrow ??$$

Workbook.

pb 3.





(6) d✓ a✓

(15) VI में सिर्फ springs → No damping $\zeta = 0$ रख देंगे।

(31) b✓

(32) Frahms B tachometer → (c)✓

(33) d✓ → Magnitude factor = Magnification factor है।

(34) 1/3

(35) a periodic → non-periodic.